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IMAGE PROPERTIES OF A SUPERCONDUCTING GROUND PLANE

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ABSTRACT

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The Green's function technique is utilized in the determination of the field distribution of an infinitely long current carrying conductor of arbitrary but constant cross section above a superconducting ground plane of finite thickness. It is assumed that the superconductor can be described by the phenomenological London equations. The integral expressions that are obtained are solved analytically for a few special cases of interest. It is shown that under certain conditions, which are often encountered in a physical system, a modified image method can be utilized in order to calculate the field distribution to within 2 percent of the computer solution. AUTHOR

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INTRODUCTION

Due to the rapid advance of technology in the area of superconductive devices for computer applications, a great deal of interest has been expressed in developing high speed switching components.[1] It has been shown that a device whose basic structure is that of a thin superconducting film above a superconducting ground plane exhibits the desired switching speed characteristics.[2,3] The basis of any electromagnetic analysis of this type has been the assumption that the return current in the ground plane is localized under the current carrying film effectively forming a strip transmission line whose characteristics are then calculated.[4,5] The previous assumption (henceforth called the Strip Line assumption) is made for geometries in which the film width is much larger than the spacing between the film and ground plane.

However, it is often necessary to determine the field distribution for geometries whose parameters do not fit the requirement needed for the Strip Line assumption. In this paper, the Green's function technique is used in deriving a general expression for the field of a conductor, which is carrying current above a superconducting ground plane. The Strip Line assumption is verified and a few cases of particular interest are examined. A modified image technique is developed that is useful for determining the field of the current carrying conductor if certain geometrical conditions, which are often encountered in practice, are satisfied.

Characterization of the Superconducting Ground Plane

It will be assumed that the properties of the ground plane can be described by the London equations

$$\vec{E} = \mu_o(\beta)^{-2} \frac{\partial \vec{J}}{\partial t} \tag{1}$$

$$\vec{H} = -(\beta)^{-2} \nabla \times \vec{J} \tag{2}$$

where β is the reciprocal London penetration depth and $\vec{B} = \mu_o \vec{H}$. (The rationalized mks system of units is used throughout this paper.) It can be shown from Eqs. (1) and (2) that in the coulomb gauge under static conditions

$$\nabla \times (\vec{A} + \mu_o \beta^{-2} \vec{J}) = 0 \tag{3a}$$

$$\nabla \cdot (\vec{A} + \mu_o \beta^{-2} \vec{J}) = 0 \tag{3b}$$

and

$$\nabla^2\vec{J} = \beta^2\vec{J} \tag{4}$$

The following statements will be considered to be valid in the following analysis (Fig. 1):

(1) $\vec{J}$ in the ground plane has only one component (in the z-direction)

(2) $\vec{J}$ in the ground plane is bounded in space (this will be justified later)

It will be assumed that the current density in S' is known. In the London gauge

$$\vec{A} = -\mu_0\beta^{-2}\vec{J} \tag{5}$$

and

$$\nabla^2\vec{A} = \beta^2\vec{A} \tag{6}$$

A straightforward argument can be used to prove that $\vec{A}$ is uniquely determined for a given total ground plane current.

Statement of Problem

Consider the situation in which a current carrying source of arbitrary cross section is placed above a superconducting ground plane (Fig. 1). A fraction $\xi = I_1/I_0$ of the current is returned through the ground plane and $(1 - \xi)$ of the current is returned through the wire above the conductor. It is assumed that there is no geometric variation in the z-direction so that $\partial/\partial z$ of the field quantities are zero.

The source current density $\vec{J}_S(x',y')$, which has only a z-component can be separated into a symmetric and antisymmetric part (with respect to y') called $\vec{J}_s$ and $\vec{J}_a$, respectively.

$$\vec{J}_s(x',y') = \vec{J}_S(x',y') + \vec{J}_S(x',-y') \tag{7a}$$

$$\vec{J}_a(x',y') = \vec{J}_S(x',y') - \vec{J}_S(x',-y') \tag{7b}$$

where

$$\int_{S'} \vec{J}_s(x',y') \ \ d\vec{S}' = I_0 \tag{8}$$

The Green's function diagram, which corresponds to Fig. 1, is shown in Fig. 2. Due to the separation defined by Eqs. (7a) and (7b) symmetric and antisymmetric pairs of delta function sources are used to generate the Green's function of the system. For simplicity consider the case for $\xi = 1$ first. Since free space can be described by the equation

$$\nabla^2\vec{A} = -\mu_0\vec{J} \tag{9}$$

the solutions to Eqs. (5) and (9) in the regions shown in Fig. 2 are as follows: ($A_{zs}(\vec{r}/\vec{r}\,')$ and $A_{za}(\vec{r}/\vec{r}\,')$ are the symmetric and antisymmetric (Green's) vector potentials, respectively).

Region 1,2:

$$\begin{Bmatrix} A_{zs}(\vec{r}/\vec{r}\,')_{1,2} \\ A_{za}(\vec{r}/\vec{r}\,')_{1,2} \end{Bmatrix} = \int_0^\infty \begin{Bmatrix} C_{1s}(k)\cos ky \\ C_{1a}(k)\sin ky \end{Bmatrix} e^{-kx}\, dk \tag{10a}$$

Region 3:

$$\begin{Bmatrix} A_{zs}(\vec{r}/\vec{r}\,')_3 \\ A_{za}(\vec{r}/\vec{r}\,')_3 \end{Bmatrix} = \int_0^\infty \begin{Bmatrix} \cos ky \\ \sin ky \end{Bmatrix} \left[\begin{Bmatrix} C_{2s}(k) \\ C_{2a}(k) \end{Bmatrix} e^{-kx} + \begin{Bmatrix} C_{3s}(k) \\ C_{3a}(k) \end{Bmatrix} e^{kx} \right] dk \tag{10b}$$

Region 4:

$$\begin{Bmatrix} A_{zs}(r/r')_4 \\ A_{za}(r/r')_4 \end{Bmatrix} = \int_0^\infty \begin{Bmatrix} \cos ky \\ \sin ky \end{Bmatrix} \left[\begin{Bmatrix} C_{4s} \\ C_{4a} \end{Bmatrix} e^{\sqrt{\beta^2+k^2}\, x} + \begin{Bmatrix} C_{5s} \\ C_{5a} \end{Bmatrix} e^{-\sqrt{\beta^2+k^2}\, x} \right] dk \tag{10c}$$

Region 5:

$$\begin{Bmatrix} A_{zs}(r/r')_5 \\ A_{za}(r/r')_5 \end{Bmatrix} = \int_0^\infty \begin{Bmatrix} C_{6s} \cos ky \\ C_{6a} \sin ky \end{Bmatrix} e^{kx}\, dk \tag{10d}$$

The solution e^{kx} was rejected for $x > x'$, and the solution e^{-kx} was rejected for $x < -d$, since it is assumed that the current density in the ground plane is bounded (in the x,y plane) and that the Green's vector potential is zero at $x,y = \infty$. This is reasonable since the ground plane in any physical situation is not infinitely wide but can indeed be very wide in comparison to the source conductor dimension and x'. A similar statement can be made regarding boundedness in the z-direction. Therefore, the boundedness statement given in the last section is justified due to the above physical argument.

In order to evaluate the C's, it is necessary to realize that the x-component of $\vec{B}$ is continuous at $x = x', 0, -d$, while the y-component of $\vec{H}$ is continuous at $x = 0, -d$ and is discontinuous at $x = x'$. The five equations describing the continuity of field are simply algebraic in form and will not be explicitly shown here. The discontinuity of H_y at $x = x'$ is not quite so obvious and will be derived here. At $x = x'$

$$H_y(\text{region } 1,2) - H_y(\text{region } 3) = \mu_0[\delta(y - y') \pm \delta(y + y')] \tag{11}$$

The signs on the right refer to the symmetric and antisymmetric cases, respectively. $\delta(y \pm y')$ is the Dirac delta function. Eq. (11) becomes

$$\int_0^\infty \left\{ [C_1(k) - C_2(k)]ke^{-kx'} + C_3(k)ke^{-kx'} \right\} \begin{Bmatrix} \cos ky \\ \sin ky \end{Bmatrix} dk$$

$$= \mu_0[\delta(y - y') \pm \delta(y + y')] \tag{12}$$

Using the Fourier integral theorem yields

$$[C_1(k) - C_2(k)]e^{-kx'} + C_3(k)^{kx'} = \frac{2\mu_o}{k\pi}\begin{Bmatrix}\cos ky' \\ \sin ky'\end{Bmatrix} \tag{13}$$

Simultaneous solution of Eq. (13) and the five continuity equations yields the values for the C's. If S is defined as cos ky' in the symmetric case and sin ky' in the antisymmetric case and

$$\left.\begin{aligned}\alpha_\pm &= k \pm \sqrt{\beta^2 + k^2} \\ \gamma_\pm &= e^{\pm\sqrt{\beta^2+k^2}\,d} \\ \delta_\pm &= 1 \pm e^{2kx'}\end{aligned}\right\} \tag{13a}$$

then

$$\left.\begin{aligned}C_1 &= \frac{S}{\Delta}\frac{2\mu_o}{\pi} e^{k(x'-d)}\left[(\delta_-)\ \frac{\alpha_+ - k}{k}\ (\gamma_+\alpha_+ + \gamma_-\alpha_-) + (\delta_+)(\gamma_-\alpha_- - \gamma_+\alpha_+)\right] \\ C_2 &= \frac{S}{\Delta}\frac{2\mu_o}{k\pi} e^{k(x'-d)}[(\alpha_-)(\alpha_+)(\gamma_+ - \gamma_-)] \\ C_3 &= \frac{S\mu_o}{k\pi} e^{-kx'} \\ C_{\substack{4\\5}} &= \pm \frac{4S\mu_o}{\Delta\pi} e^{k(x'-d)}\alpha_\pm\gamma_\pm \\ C_6 &= \frac{-S}{\Delta}\frac{8\mu_o}{\pi} e^{kx'}(\alpha_+ - k)\end{aligned}\right\} \tag{13b}$$

where

$$\Delta = +2e^{k(2x'-d)}\left[(\gamma_+)(\alpha_+)^2 - (\gamma_-)(\alpha_-)^2\right]$$

Once the Green's potentials have been found, it is a simple matter to express the vector potential for the configuration shown in Fig. 1

(remembering that the case for $\xi = 1$ is under consideration):

Regions I, II, IV, V, VI:

$$A_z(x,y) = \iint\limits_{\frac{1}{2}S'} \left[J_s(x',y')A_{zs}(\vec{r}/\vec{r}\,') + J_a(x'y')A_{za}(\vec{r}/\vec{r}\,')\right]dx'\,dy' \tag{14}$$

($\frac{1}{2}$ S' refers to an integration over the source area defined by $y' \geq 0$) where the primed integration is over the source cross section. $A_{zs}(\vec{r}/\vec{r}\,')$ and $A_{za}(\vec{r}/\vec{r}\,')$ are the appropriate vector potentials for the region under consideration.

Region III:

$$A_z(x,y) = \int_{S_1}\!\!\int \left[J_s(x',y')A_{zs}(\vec{r}/\vec{r}\,')_1 + J_a(x',y')A_{za}(\vec{r}/\vec{r}\,')_1\right]dx'\,dy'$$

$$+ \int_{S_2}\!\!\int \left[J_s(x',y')A_{zs}(\vec{r}/\vec{r}\,')_3 + J_a(x',y')A_{za}(\vec{r}\,'/\vec{r}\,')_3\right]dx'\,dy' \tag{15}$$

where S_1 is the portion of the source region in which

$$x < x' \qquad y' \geq 0 \tag{16a}$$

and S_2 is that portion in which

$$x > x' \qquad y' \geq 0 \tag{16b}$$

In the above equations, $\vec{r}\,'$ and $\vec{r}$ are the source and field points, respectively.

It can now be shown that the total current in the ground plane is $-I_0$. From Eqs. (5) and (14) it is seen that the current in the ground plane is

$$I = -\frac{\beta^2}{\mu_0} \iint_{S_{GP}} \iint_{\frac{1}{2}S'} \left[J_s(x'y') A_{zs}(\vec{r}/\vec{r}')_4 \right] dx'\ dy'\ dx\ dy \tag{17}$$

where the unprimed coordinates refer to a point in the ground plane. (It can be shown by direct integration that the antisymmetric current density does not contribute to the net ground plane current.) Integrating over y first yields

$$I = -\frac{\beta^2}{\mu_0} 2\pi \int_{-d}^{0} \iint_{\frac{1}{2}S'} \int_{0}^{\infty} \delta(k) J_s(x'y') \left[C_{4s}\ e^{\sqrt{\beta^2+k^2}\,x} + C_{5s}\ e^{\sqrt{\beta^2+k^2}\,x} \right] dx'\ dy'\ dx \tag{18}$$

Integrating over k and x, respectively, yields

$$I = -2 \iint_{\frac{1}{2}S'} J_s(x'y') dx'\ dy' = -I_0 \tag{19}$$

which completes the proof.

Partial Current Return in the Ground Plane $(0 \leq \xi < 1)$

This case can be treated as the superposition of two situations. In the first situation, all the source current I_0 is returned through the ground plane. In the second situation there is no source conductor, and the return wire carries current $(I_1 - I_0)$, which is returned through the ground plane.

Using this superposition the Green's vector potential can be found directly by considering the case for $\xi = 1$.

Define constants D(k) that are related to C(k) by the relations

$$D_s(k) = -(1 - \xi)C_s(k)/2 \qquad (20a)$$

(and replace x' in the $C_s(k)$ expressions by x")

$$D_a(k) = 0 \qquad (20b)$$

Thus, the Green's potentials are:

Region 1:

$$A_z(\vec{r}/\vec{r}\,')_1 = A_z(\vec{r}/\vec{r}\,')_1\Big|_{\xi=1} + \int_0^\infty D_1(k)\cos ky\; e^{-kx}\, dk \qquad (21a)$$

Region 2:

$$A_z(\vec{r}/\vec{r}\,')_2 = A_z(\vec{r}/\vec{r}\,')_1\Big|_{\xi=1} + \int_0^\infty \cos ky\left[D_2(k)e^{-kx} + D_3(k)e^{kx}\right]dk \qquad (21b)$$

Region 3:

$$A_z(\vec{r}/\vec{r}\,')_3 = A_z(r/r')_3\Big|_{\xi=1} + \int_0^\infty \cos ky\left[D_2(k)e^{-kx} + D_3(k)e^{kx}\right]dk \qquad (21c)$$

Region 4:

$$A_z(\vec{r}/\vec{r}\,')_4 = A_z(\vec{r}/\vec{r})_4\Big|_{\xi=1} + \int_0^\infty \cos ky\left[D_4(k)e^{\sqrt{\beta^2+k^2}\,x} + D_5(k)e^{-\sqrt{\beta^2+k^2}\,x}\right]dx \qquad (21d)$$

Region 5:

$$A_z(\vec{r}/\vec{r}\,')_5 = A_z(\vec{r}/\vec{r}\,')_5\Big|_{\xi=1} + \int_0^\infty D_6(k)\cos ky\; e^{kx}\, dk \qquad (21e)$$

For many cases of practical interest, the return wire is far from the ground plane and the source conductor. It can be verified that

$$\lim_{x''\to\infty} D_n(k) = -C(0)(1 - \xi)\,\frac{\delta(k)}{\delta(0)} \qquad n = 2, \ldots, 6 \tag{22a}$$

Therefore,

$$\lim_{x''\to\infty} A_z(\vec{r}/\vec{r}\,')_n\Big|_{0\le\xi<1} = A_z(\vec{r}/\vec{r}\,')\Big|_{\xi=1} \qquad n = 2, \ldots, 6 \tag{22b}$$

The net current in the ground plane can be found in a manner identical to Eqs. (17) to (19) and can be shown to be $-I_1$. This means that although the ground plane only returns a fraction of the source current, the vector potential is identical to the case for $\xi = 1$ if the return wire is far from the source and the ground plane. If one remembers the superposition that is the basis of Eqs. (21(a-e)), it is apparent that if the ground plane carries a current $-I_1$, $-I_0$ of the current follows the distribution given by Eq. (17), while $(I_0 - I_1)$ is uniformly distributed over the entire ground plane cross section. The current density of the uniform distribution is infinitely small so that it does not contribute to the vector potential, however, its integral over the cross section of the ground plane is still $(I_0 - I_1)$. It should also be noted that Eq. (22a) is valid only if I_0 is finite. A geometry that violates this restriction will be examined below.

The above discussion is not as remote as it may seem. Consider a ground plane of finite width W_g. Assume that the source conductor has dimensions small compared to W_g and is close to the ground plane surface. Also assume that the return wire is far from the source conductor and ground plane. However, stipulate that the perpendicular distance from the return wire to the ground plane is also much less than W_g. For this case, Eq. (22b) is satisfied in the neighborhood of the source conductor. For certain geometries (discussed later) the fields in the neighborhood of the source are very large in comparison to the fields elsewhere so that Eq. (22b) can be used to advantage in calculating the inductance of the structure.

The Green's vector potential is now completely specified for arbitrary ξ so that it is now appropriate to examine a few source conductor geometries of general interest (it will be assumed that the return wire is essentially at infinity for the remainder of the paper).

Special Cases of Interest

Case 1:

$$\vec{J}_S(x',y') = J_L\delta(x' - a)$$

This source distribution violates the requirement that I_0 be finite so that Eq. (22b) cannot be applied here. Although the fields from this distribution can be more easily obtained by a one-dimensional analysis, they can also be obtained from the above equations to demonstrate the validity of the analysis. Using Eq. (14) with Eqs. (21(a-e)) for the Green's vector potential and performing the curl operation yield the following relationships

$$x > a \quad H_y = J_L(1 - \xi)/2 \qquad (23a)$$

$$0 < x < a \quad H_y = -J_L(1 + \xi)/2 \qquad (23b)$$

$$-d < x < 0 \quad H_y = \frac{J_L(1+\xi)}{2}\left\{\left[\frac{(1-\xi) - (1+\xi)\cosh\beta d}{(1+\xi)\sinh\beta d}\right]\sinh\beta x - \cosh\beta x\right\} \qquad (23c)$$

$$x < -d \quad H_y = -J_L(1 - \xi)/2 \qquad (23d)$$

Case 2:

$$J_S(x',y') = \begin{cases} J_L\delta(x' - a) & |y'| < W \\ 0 & |y'| > W \end{cases} \tag{24}$$

The current per unit thickness in the ground plane, which is contained in a width $2l$, is

$$J_l(x) = -\frac{\beta^2}{\mu_o}\int_{-l}^{l}\int_0^{\infty}\int_0^{W}\int_{-\infty}^{\infty} J_L\delta(x' - a)\cos ky\left[C_{4s}(k)e^{\sqrt{\beta^2+k^2}\,x}\right.$$

$$\left. + C_{5s}(k)e^{-\sqrt{\beta^2+k^2}\,x}\right] dx'\, dy'\, dk\, dy \tag{25}$$

Integration with respect to x', y', and y, respectively, yields

$$J_l(x) = -\frac{2\beta^2}{\mu_o}\int_0^{\infty} J_L\left[\frac{C_{4s}(k,x' = a)}{\cos ky'}\, e^{\sqrt{\beta^2+k^2}\,x}\right.$$

$$\left. + \frac{C_{5s}(k,x' = a)}{\cos ky'}\, e^{-\sqrt{\beta^2+k^2}\,x}\right]\frac{\sin kl}{k}\,\frac{\sin kW}{k}\, dk \tag{26}$$

Introducing the change of variables $k' = k\,d$ yields for the case $W \gg \beta^{-1},a$

$$J_l(x) \approx \frac{-4\beta W l J_L}{d\pi}\left[\frac{e^{\beta(d+x)} + e^{-\beta(d+x)}}{e^{+\beta d} - e^{-\beta d}}\right]\int_0^{\infty}\frac{\sin\frac{k' l}{d}}{k'\,\frac{l}{d}}\;\frac{\sin k'\,\frac{W}{d}}{k'\,\frac{W}{d}}\, dk' \tag{27}$$

The current density is given by

$$\frac{dJ_l(x)}{d(2l)}$$

so that for $l < W$

$$J_z(x) \approx -\beta J_L \left[\frac{e^{\beta(d+x)} - e^{-\beta(d+x)}}{e^{+\beta d} - e^{-\beta d}} \right] \tag{28}$$

$$l > W$$

$$J_z(x) \approx 0 \tag{29}$$

which verifies the Strip Line assumption discussed in the introduction.

Modified Image Technique

There is a large class of problems that can be solved without resorting to manipulations such as were shown in case 1 and 2 by using a modified image technique that will now be discussed. For simplicity, consider the Green's vector potential that is generated by a single delta function source of unit strength inside the source conductor at $x' = a$. There will be no loss in generality by taking $y' = 0$.

If d_1 (see Fig. 1) is much larger than β^{-1} then $a/\beta^{-1} \gg 1$, since a is the x-coordinate of an arbitrary delta function source inside the source conductor. If $a/\beta^{-1} \gg 1$ the Green's current density in the ground plane is (Eqs. (5) and (14))*

$$J_z(\vec{r}/\vec{r}\,') \approx -\frac{1}{\pi}\frac{\beta}{d}\,\frac{\cosh\left[\left(\frac{x}{d}+1\right)\beta d\right]}{\sinh \beta d}\int_0^\infty \cos\frac{ky}{d}\, e^{-ka/d}\, dk \tag{30}$$

which reduces to

$$J_z(\vec{r}/\vec{r}\,') \approx -\frac{1}{\pi}\,\frac{a}{a^2+y^2}\,\beta\,\frac{\cosh\left(\frac{x}{d}+1\right)\beta d}{\sinh \beta d} \tag{31}$$

*A factor of 1/2 must be introduced into equation (14), since here a single delta function source is being considered (rather than a symmetrical pair).

The factor $-a/\pi(a^2 + y^2)$ is the current per unit width at the $x = 0$ surface of the ground plane which is found by replacing the ground plane by the image of the delta function source. Since Eq. (31) is valid for every delta function source that makes up the source conductor the following general rule can be established. Given a source conductor of arbitrary cross section with the specification that $d_1 \gg \beta^{-1}$ the current density in the ground plane can be found by first finding the current per unit width at $x = 0$ by using the method of images and then multiplying $J_z(x = 0)$ by $\beta \cosh\left[\left(\frac{x}{d} + 1\right)\beta d\right] \Big/ \sinh \beta d$.

Equation (31) reduces to the perfect conductivity limit $(\beta^{-1} \to 0)$ as can be seen by considering the following:

$$\lim_{\beta\to\infty} -\frac{1}{\pi}\,\beta e^{\beta x}\,\frac{a}{a^2 + y^2} \tag{32a}$$

It is interesting to note that

$$\lim_{\beta\to\infty} \beta e^{\beta x} = \begin{cases} \infty & x = 0 \\ 0 & x < 0 \end{cases} \tag{32b}$$

and

$$\lim_{\beta\to\infty} \int_{-d}^{0} \beta e^{\beta x}\, dx = 1 \tag{32c}$$

Thus, for $x \leq 0$

$$\lim_{\beta\to\infty} \beta e^{\beta x} = \delta(x) \tag{33}$$

and

$$J_z(\vec{r}/\vec{r}\,') = -\frac{1}{\pi}\,\frac{a}{a^2 + y^2}\,\delta(x) \tag{34}$$

which is the correct result if the ground plane is a perfect conductor.

It is now instructive to examine the numerical solution to the Green's current density in the ground plane for the case of a unit delta function source at $x' = a$ and $y' = 0$. The dimensionless expression for the current density (from Eqs. (5) and (14)) is

$$J_z(x,y)d^2 = -\int_0^{\infty} dk\, \frac{(\beta d)^2}{\pi} \cos\left(\frac{y}{d}\, k\right) \left[A_1 e^{\left(\frac{x}{d}+1\right)\sqrt{(\beta d)^2+k^2}} + A_2 e^{-\left(\frac{x}{d}+1\right)\sqrt{(\beta d)^2+k^2}}\right] \tag{35a}$$

where

$$\left\{\begin{matrix} A_1 \\ \\ A_2 \end{matrix}\right\} = \pm \frac{e^{-(a/d)k}\left[k \pm \sqrt{(\beta d)^2 + k^2}\right]}{e^{\sqrt{(\beta d)^2+k^2}}\left[k + \sqrt{(\beta d)^2 + k^2}\right]^2 - e^{-\sqrt{(\beta d)^2+k^2}}\left[k - \sqrt{(\beta d)^2 + k^2}\right]^2} \tag{35b}$$

Although the left side of Eq. (35a) does not appear to be dimensionless, it should be remembered that a unit strength source is under consideration. Equation (35) was solved by a modified Simpson's rule using an IBM 7090 computer. It was found that the integrand converged rapidly to zero so that infinity could be replaced by a finite number N (the highest N that was used was 81). The results are given in Figs. 3 to 6. It should be noted by comparing Eq. (31) with Figs. 3 and 5 that the modified image technique developed in this paper is valid to within 2 percent. The numerical results for the Green's current density can be used to evaluate the current density in the ground plane for an arbitrary source conductor by using approximation techniques for cases in which the modified image technique is not valid.

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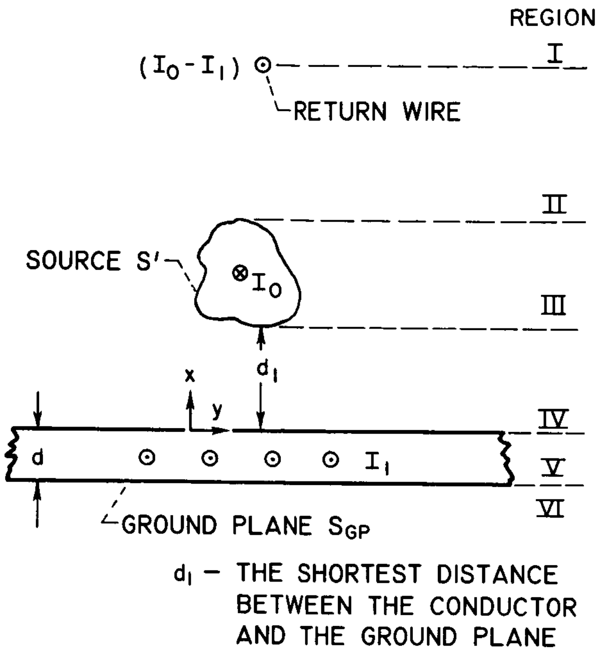

Fig. 1. - Current carrying conductor above superconducting ground plane.

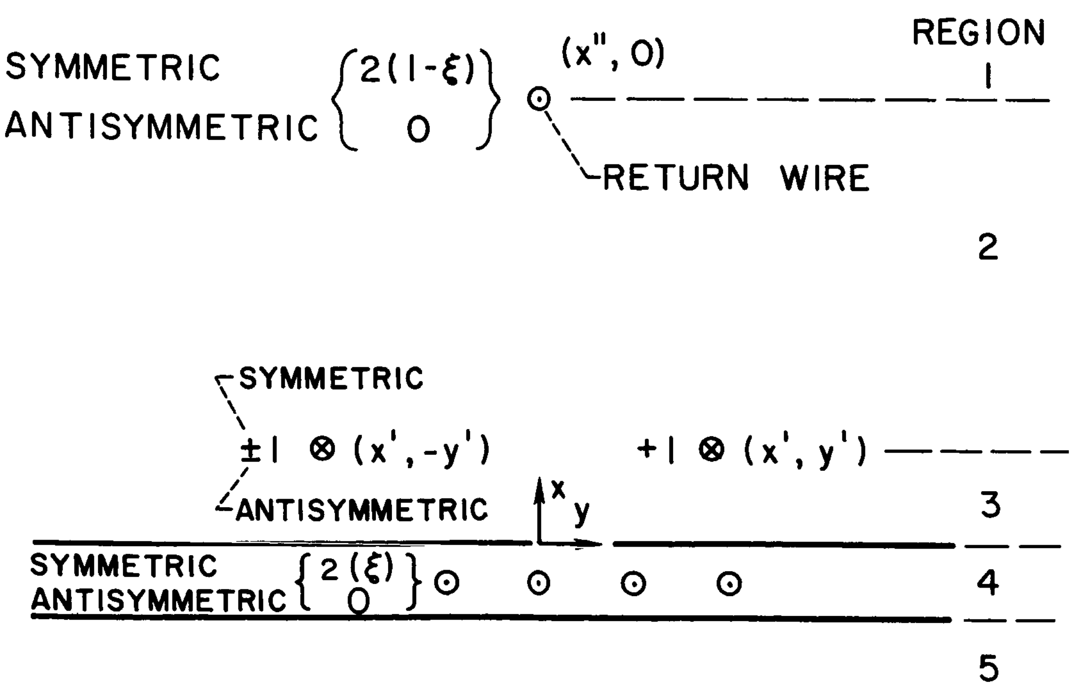

Fig. 2. - Green's function diagram.

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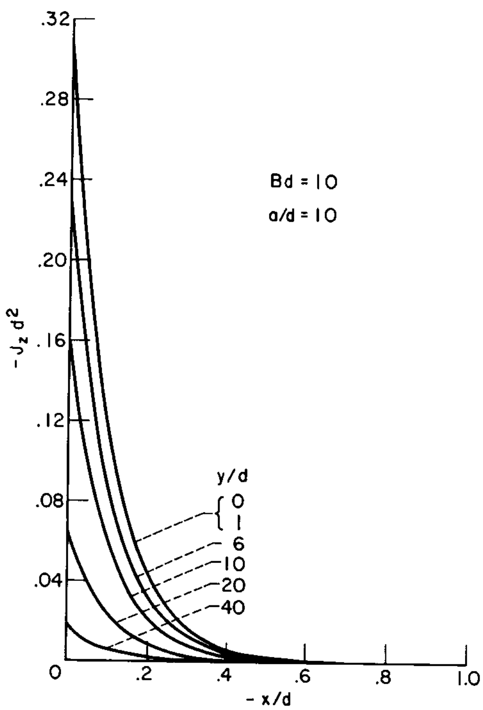

Fig. 3. - Ground plane current density.

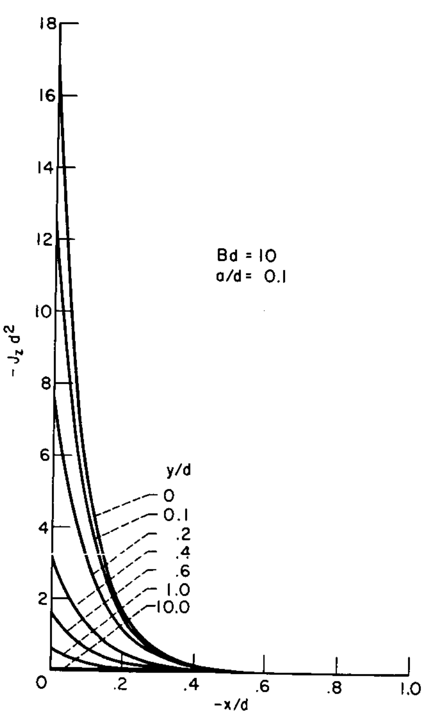

Fig. 4. - Ground plane current density.

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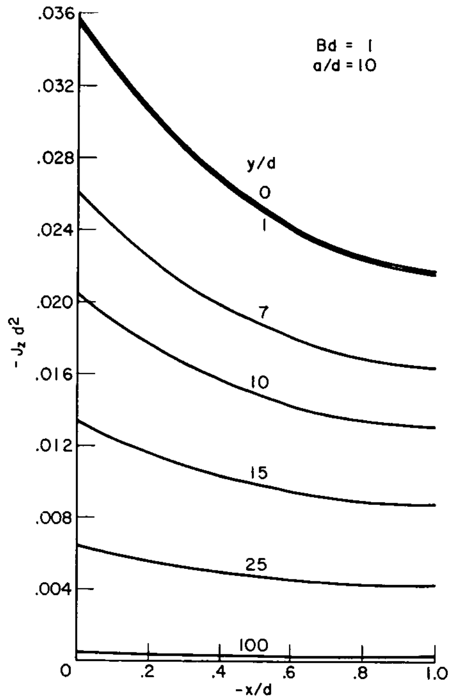

Fig. 5. - Ground plane current density.

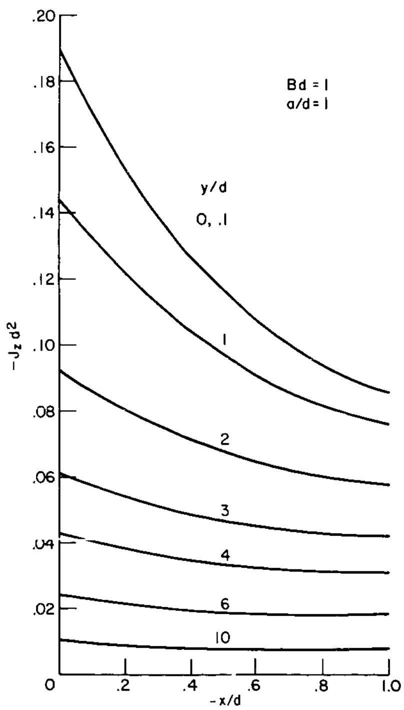

Fig. 6. - Ground plane current density.

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